Characterizing Investor Expectations for Assets with Varying Risk

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Abstract

How do financial market investors form expectations about assets with different risk characteristics? We examine this question using Euro-area yield curves for AAA-rated and AAA-with-other bonds. Investors' conditional forecasts about the yield curves for different assets, at various forecasting horizons, are modeled using a VAR model with time-varying parameters. Two processes are assumed for the evolution of these parameters: a constant-gain learning model and a new endogenous learning technique proposed here. Both these algorithms allow investors to account for structural changes in the data. The endogenous learning mechanism also allows investors to compensate for large deviations in observed coefficients used for forecasting, relative to past data. Daily data is used to estimate the gain parameters for the learning algorithms, and we find that these gains vary across asset types, implying investors form conditional expectations differently for assets with differential risks. For 2005-2015, the investors' conditional forecasts for the AAA-rated bonds are better described using the endogenous learning mechanism, implying that investors with lower risk preferences are more sensitive to large deviations in the data.

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1 Introduction

Expectations of investors about the cross-section of yields are important for policy makers and financial markets: forecasts of the Treasury yield curves are central for the transmission of monetary policy actions from the short end of the yield curve to the long end; conditional expectations about yields on riskier assets affect borrowing costs for a variety of firms and investors. The importance of expectations formation has been widely analyzed. Hommes (2006) presents survey evidence about the rational expectations paradigm may not be fully representative of expectations formation in financial markets: the excess volatility in stock prices and survey expectations of professional forecasters suggest that different forecasting strategies are being used. However, the literature on estimating these expectations from the data is still relatively underdeveloped. Some of the new approaches used to model expectations formation in the boundedly rational approach are discussed in Hommes (2013), such as heterogeneous agent-based models and evolutionary learning.

In this paper, we propose to estimate and characterize the expectations formation process of financial investors. We are specifically interested in exploring how investors form beliefs for asset yields with distinct risk profiles, over different maturities. Traditionally, rational expectations has been the dominant paradigm used for modeling investor beliefs for assets, irrespective of their risk characteristics. However, an expanding literature finds that the use of rational expectations may be inadequate. A wide range of survey data from professional forecasters shows systematic variations in forecasting errors[3]; this is counter to the rational expectations hypothesis for such investors. For example, Gourinchas and Tornell (2004) show that the foreign exchange forward premium puzzle can be shown to arise from systematic distortions in investor beliefs about interest rates and document this distortion using survey data from G-7 countries. Bacchetta, Mertens and Van Wincoop (2009) investigate the link between predictability of excess returns and expectational errors in the stock market, foreign exchange and bond and money markets, using data on survey expectations of market participants in various countries. The authors find that in markets with predictable excess return patterns, expectational errors of excess returns are also predictable, with same signs and similar magnitudes.

We use a novel European dataset to characterize the conditional expectations of investors.

[3] This is true for forecasts of interest rates as well as macroeconomic variables such as GDP and inflation.

A unique feature of the Euro-area yield curve data is that two types of yield curves are estimated: yields for AAA-rated only bonds, and yields on bonds with AAA- and other types of bonds. This enables us to distinguish between the expectations formation process for bonds with varying risk attributes. We ask whether investors form conditional forecasts of riskless or AAA-rated assets in the same way as for assets with higher risk. Our analysis also examines whether the beliefs of investors are time-varying, over the other characteristics of maturity and forecast horizons.

We employ the following strategy: estimates of the Euro-area yield curves (based on a latent factor model) are obtained from the European Central Bank (ECB). Using this factor model, implied conditional expectations of yields (and associated latent factors) are formed using a vector auto-regressive (VAR) model of the latent factors. We minimize the root mean squared errors (RMSE) of the implied yield forecasts relative to observed yields to reveal which expectation formation process would have achieved the best forecasting performance.

The intuition for our strategy can be described as follows. As a benchmark, consider this framework with constant coefficients. A constant coefficients model restricts the investors to place identical weights on past information while forecasting the short and long asset yields. The model also implies that the investors must be using constant coefficients to form expectations over different forecasting horizons. Thus, it does not allow investors to endogenously adapt to any structural breaks that they might perceive in the evolution of the average yields, or the yield curve slope. This seems undesirable from a practical point of view, particularly during periods of high perceived structural change.

Therefore, we explore alternative specifications for the formation of conditional forecasts of the yield curve factors, and subsequent yields. Theoretical analyses, such as Piazzesi, Salomao and Schneider (2015) and Sinha (2015), incorporate adaptive learning into the expectations formation of optimizing agents in models of the yield curve. The implied term structures are more successful at matching the properties of the empirical yield curve, relative to models with time-invariant beliefs. A class of adaptive learning models is also considered here for expectations formation: constant gain learning and an endogenous learning algorithm. The main innovation is that investors are now allowed to vary the weights they place on past information about yields; they are also able to change these weights in response to large and persistent deviations observed in the yield curve factors.

Our empirical strategy allows us to estimate the gain parameters from the data. While these are conditional on the forecasting model used, to our knowledge, these provide the first estimates in the literature about how investors form expectations about different types of assets. We find that over our sample period (between September 2005 and June 2015), the performance of the constant gain algorithm is frequently overtaken by the endogenous learning model for the safest (only AAA-rated) assets. This suggests that investors, in fact, use models with time-varying coefficients to form their conditional forecasts. They also adjust the weights placed on past observations when large deviations in the coefficients are observed. These adjustments in conditional forecasts of yields may also potentially effect the holdings of safe assets by investors.

This paper is organized as follows: section two gives a brief overview of the literature. The factor model for the nominal yield curve is presented in section three. Section four discusses the different learning mechanisms and section five presents the numerical results. Section six concludes.

2 Related Literature

Time-varying beliefs have been widely incorporated in partial and general equilibrium models of asset prices to match characteristics of the data. Branch and Evans (2010) use a model of recursive least-squares learning to explain asset pricing dynamics observed in U.S. data, such as excess returns. The authors also show the existence of multiple equilibria, and that under optimal forecasting rules, switching may occur between these equilibria. Laubach, Tetlow and Williams (2007) allow investors to re-estimate the parameters of their term structure model based on incoming data. In Branch and Evans (2011), the authors show that when agents learn about the riskiness of stocks, price bubbles and ensuing crashes can be generated. Piazzesi, Salomao and Schneider (2015) decompose expected excess returns into the returns implied by the statistical VAR model and survey expectations, used as an approximation for subjective investor expectations. Survey expectations are found to be significantly more volatile compared to model implied returns. The authors use constant-gain learning to describe these expectations, and the excess returns implied by the learning model capture movements in the empirical data better. The common theme of these analyses is the incorporation of subjective beliefs in explaining characteristics of the empirical term structure. The distinguishing feature of our analysis is we use the term structure data to estimate the process that produces the best forecasts at different forecast horizons and maturities.

Endogenous learning algorithms have been previously introduced in the literature by Marcet and Nicolini (2003) and Milani (2014). In the former analysis, the authors incorporate bounded rationality in a monetary model; the agents switch between using a constant gain and a decreasing gain algorithm. They are successfully able to explain the recurrent hyperinflation across different countries during the 1980s. In Milani (2014), the agents switch between gains based on the historical average of the forecasting errors, instead of a fixed value. Gaus (2014) proposes a variant of the endogenous gain learning mechanism, in which the agents adjust the gain coefficient in response to the deviations in observed coefficients. Kostyshyna (2012) develops an adaptive step-size algorithm to model time-varying learning in the context of hyperinflations.

Finally, this paper hypothesizes that economic agents form expectations differently about assets with varying risk characteristics. This may be due to their individual preferences or the costs associated with holding these assets. Verrecchia (1982) uses a model of information acquisition with heterogeneous traders to show that learning from costly private information and freely available asset prices affects the distribution of traders' risk preferences.

3 Factor Model for the Euro-area Nominal Yield Curve

The ECB provides estimates of the yield curves associated with different types of bonds. Daily estimates of the zero-coupon yield curves are available from September 6, 2004 on the ECB's website. The bond and price information is provided by EuroMTS Ltd. and ratings are provided by Fitch Ratings. There are six criteria used to select bonds: (1) Bonds issued in Euros by Euro area central governments (European System of Accounts 1995: sector code S.1311); (2) Bonds with an outstanding amount of atleast Euro 5 Billion; (3) Bonds with special features, including those with specific institutional arrangements are excluded; (4) Fixed coupon bonds with a finite maturity and zero coupon bonds are included, while variable coupon bonds and perpetual bonds are excluded; (5) Actively traded central government bonds with a maximum bid-ask spread per quote of 3 basis points are selected; (6) Residual maturity brackets are fixed as ranging from three months to thirty years. Other characteristics are available on the ECB website[4].

The two datasets for which the yield curves are generated are: the first containing only AAA-rated Euro-area central government bonds (most favorable credit risk assessment), and the other containing other government bonds, in addition to the AAA-bonds. We use these yield curves to characterize the formation of expectations by investors for asset portfolios two different risk profiles.

Both yield curves are modeled using the Nelson-Siegel-Svensson approach:

$$
\begin{aligned}
y_t^n &= \beta_0 + \beta_1 \frac{1-\exp\left(\frac{-n}{\tau_1}\right)}{\frac{n}{\tau_1}} + \beta_2 \left[\frac{1-\exp\left(\frac{-n}{\tau_1}\right)}{\frac{n}{\tau_1}} - \exp\left(\frac{-n}{\tau_1}\right)\right] \qquad (1) \\
&\quad + \beta_3 \left[\frac{1-\exp\left(\frac{-n}{\tau_2}\right)}{\frac{n}{\tau_2}} - \exp\left(\frac{-n}{\tau_2}\right)\right].
\end{aligned}
$$

Here y_t^n is the zero-coupon yield of maturity n months at time t, β_0 approximates the level of the yield curve, β_1 approximates its slope, β_2 the curvature and β_3 the convexity of the curve. The latter captures the hump in the yield curve at longer maturities (20 years or more). When $\beta_3 = 0$, the specification in (1) reduces to the Nelson-Siegel (1987) form. The parameters in (1), which are $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1$ and τ_2 are estimated using maximum likelihood by minimizing the sum of squared deviations between the actual Treasury security prices and the predicted prices.[5]

To construct yield forecasts using the representation in (1), it must be amended with a process for the evolution of the factors. Diebold and Li (2006) and Aruoba, Diebold and Rudebusch (2006) specify the two-step estimation of yields and factors:

$$
\begin{aligned}
\mathbf{y}_t &= \mathbf{X}_t \boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t \qquad (2a) \\
\boldsymbol{\beta}_t &= \boldsymbol{\mu} + \Phi \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t. \qquad (2b)
\end{aligned}
$$

[4] https://www.ecb.europa.eu/stats/money/yc/html/technical_notes.pdf

[5] The prices are weighted by the inverse of the duration of the securities. Underlying Treasury security prices in the Gürkaynak, Sack and Wright estimation are obtained from CRSP (for prices from 1961 - 1987), and from the Federal Reserve Bank of New York after 1987.

Here $\mathbf{y}_t$ is the 3×1 vector of yields, $\mathbf{X}_t$ is a 4×1 vector of the regressors in (1)[6], $\boldsymbol{\beta}_t$ is a 4×1 vector of the factors, μ is the intercept and Φ denotes the dependence of the factors on past values. We will consider this as the benchmark model for factor evolution. The variance-covariance matrices given by:

$$var(\varepsilon_t) = H = \begin{pmatrix} \sigma_1^2 & 0 & 0 \\ ... & ... & ... \\ 0 & 0 & \sigma_n^2 \end{pmatrix}; \; var(\eta_t) = Q = \begin{pmatrix} \omega_{11}^2 & \omega_{12}^2 & \omega_{13}^2 \\ ... & ... & ... \\ \omega_{n1}^2 & \omega_{n2}^2 & \omega_{n3}^2 \end{pmatrix}. \tag{3}$$

The factor errors are assumed to be distributed as a normal, with mean zero.[7]

3.1 Properties of the Fitted Yield Curves

The fitted yield curves for the AAA- and All-rated assets are shown in figure 1. The break in the yield series in evident from the start of the financial crisis: there is a significant deviation in the yields on riskless and risky assets. Table 1 shows the moments of the term structure of yields across two sample periods. Both types of yields show an increase in the standard deviation after January 2008. We also observe a rise in average yields across the maturity structure between 2008-2015; before this, the averages across AAA- and All-bond yields are similar.

4 Construction of Yield Forecasts

In order to construct yield forecasts using a model, investors are assumed to use the term structure model in (2). This requires forecasts of the factors, $\boldsymbol{\beta}_t$. If investors use the constant-coefficients model for the factors in (2b), then the forecasts are determined as:

$$E_t\hat{\boldsymbol{\beta}}_{t+h} = \left[I_3 - \hat{\Phi}^h\right]\left[I_3 - \hat{\Phi}\right]^{-1}\boldsymbol{\mu} + \hat{\Phi}^h\boldsymbol{\beta}_t. \tag{4}$$

However, this process does not allow for the investors to account for any structural changes in the data. Since our time-period covers the financial crisis of 2007 and its aftermath, this

[6]Since the parameters τ_1 and τ_2 are jointly estimated using the maximum likelihood approach, the X_t vector is time-varying.

[7]In the estimation, the cross covariances in η_t are set to zero.

would not be a valid exercise. Therefore, to allow investors to account for structural change in the underlying data, we estimate a time-varying parameters model for the factors.

Under this framework, we assume that at time t, the agents update their estimates of the parameters (μ, Φ) as new information on yields and implied latent factors becomes available. The timing is as follows: at time t, the estimates of $(\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ are derived using maximum likelihood estimation. To construct forecasts of the yields at one-, three- and six-month horizons, the investors use the updating processes described below to determine (μ_t, Φ_t). Once the parameters (μ_t, Φ_t) are estimated, they are used for constructing the conditional yield forecasts. At time $t+1$ the process is repeated, and updated estimates of (μ_{t+1}, Φ_{t+1}) are used to construct the forecasts of yields and corresponding forecast errors.

Since the parameters (μ, Φ) can now be updated (in contrast to remaining constant as in (2b)), the factor process is represented using a time-varying VAR model:

$$\boldsymbol{\beta}_t = \boldsymbol{\mu}_{t-1} + \Phi_{t-1}\boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t. \tag{5}$$

For each factor $\beta_i, i \in \{0, 1, 2, 3\}$, the coefficients $\Omega_{i,t} = \left(\mu_{i,t}, \Phi_{i,t}\right)$ are updated as:

$$\begin{aligned} \begin{pmatrix} \mu_{i,t} \\ \phi_{i,t} \end{pmatrix} &= \begin{pmatrix} \mu_{i,t-1} \\ \phi_{i,t-1} \end{pmatrix} + g_{i,t} R_{i,t-h}^{-1} q_{i,t-h} \left[\beta_{i,t} - \begin{pmatrix} \mu_{i,t-1} \\ \phi_{i,t-1} \end{pmatrix}' q_{i,t-h} \right] \\ R_{i,t} &= R_{i,t-1} + g_{i,t} \left[q_{i,t-h} q_{i,t-h}' - R_{i,t-1} \right] \end{aligned} \tag{6}$$

where $q_{i,t-1} = \left(1, \beta_{i,t}\right)_{t=0}^{\prime t-1}$, $g_{i,t}$ is a 5×5 diagonal matrix of the weights assigned by investors to the forecast errors made for $\left(\mu_{i,t}, \Phi_{i,t}\right)$, and $\beta_{i,t}$ is the latent factor derived at time t using the maximum likelihood procedure. $R_{i,t}$ is the covariance matrix. Finally, the forecasts of the yields are given by:

$$\begin{aligned} E_t \mathbf{y}_{t+h} &= \mathbf{X}_t E_t \hat{\boldsymbol{\beta}}_{t+h} \\ E_t \hat{\boldsymbol{\beta}}_{t+h} &= \left[I_3 - \hat{\Phi}_{t-1}^h \right] \left[I_3 - \hat{\Phi}_{t-1} \right]^{-1} \boldsymbol{\mu}_{t-1} + \hat{\Phi}_{t-1}^h \boldsymbol{\beta}_t. \end{aligned} \tag{7}$$

We make the assumption that while making conditional forecasts at time t, the investors do not allow for the possibility that they will revise their estimates of (μ, Φ) .This is the anticipated utility assumption (Kreps, 1988).[8] In the following sections, we show two schemes which are used to determine $g_{i,t}$.[9]

4.1 Constant gain learning

With constant gain learning (CGL), the gain matrix g_i is a 5×5 where all the elements along the diagonal are identical and remain constant over time. CGL has been a widely used method for characterizing the expectations formation for optimizing agents in macroeconomics and finance. Milani (2011) uses survey data on expectations to estimate a New-Keynesian model with constant gain learning, and finds that for the U.S. business cycle, expectations shocks regarding future real activity account for approxiately half of the business cycle fluctuations. Branch and Evans (2011) use constant gain learning in an asset pricing model to show that bubbles and crashes can emerge as endogenous responses to the fundamentals which drive asset prices. In contrast to the constant-coefficients model, investors can now allow for structural changes in the data they are forecasting, by placing an exponentially decaying weight on the history of observations. However, while CGL has been widely used, it does not allow them to modify the weights they place on past data, in case they observe actual data realizations that are significantly different. That is, at any point in time, the agents will continue to place the same weight on an observation n quarters ago that they did before.

4.2 Endogenous gain learning

Under endogenous gain learning, EGL hereafter, the investors continue to use the law of motion for the factors in (5), along with the updating equation in (6). However, while the gain matrix is still diagonal, the diagonal elements are not held fixed for the entire sample. The innovation in this learning algorithm, in contrast to CGL is that in time periods during which agents observe large deviations in the realization of coefficients, they are able to adjust

[8]For the estimation exercise, $h = 1$ for the 1- and 3-month horizons. The h-value is interpreted as signifying the forecasts for these different horizons. This assumption is made for numerical reasons: the wide variations in the factor time series imply that the eigenvalues are close to a unit root. In this case, the value of $\hat{\Phi}^h$ becomes explosive in case of $h > 1$.

[9]If the gain $g_t = 0$, then the parameters remain constant, and the forecasts of the factors will be constructed as in (4).

the weight placed on past observations upwards or downwards.

Formally, under endogenous learning, the gain switches according to the specification below:

$$g_{ij,t} = \bar{g}_i \ + \bar{g}_i^{sf} \frac{\exp\left(\frac{\Omega_{ij,t}-\bar{\Omega}_i^k}{\sigma_{\Omega_{ij}}}\right)}{1+\exp\left(\frac{\Omega_{ij,t}-\bar{\Omega}_i^k}{\sigma_{\Omega_{ij}}}\right)}. \tag{8}$$

The notation is as follows: for factor β_i, $i \in \{0,1,2,3\}$, $g_{ij,t}$ is the jth element on the diagonal of g_i. $\bar{g}_i$ is the gain for β_i and $\bar{g}_i^{sf}$ is the scaling factor used by investors to adjust their gain parameter to deviation in the Ω_{ij} coefficient matrix. $\bar{\Omega}^k$ is the average of the k most recent coefficients and $\sigma_{\Omega_{ij}}$ is the standard deviation of these k coefficients for the $ij-th$ element. In this form of learning, if the recent coefficient estimate $(\Omega_{ij,t})$ is close to the mean $(\bar{\Omega}_{ij}^k)$, then $g_{ij,t} = \bar{g}_i$. However, as the realization of $\Omega_{ij,t}$ diverges from $\bar{\Omega}_{ij}^k$, the gain approaches $\bar{g}_i + \bar{g}_i^{sf}$. We place the following constraints on the endogenous gain parameters, $0 < \bar{g}_i < 1$, $|\bar{g}_i^{sf}| < 1$ and $\bar{g}_i + \bar{g}_i^{sf} < 1$, so that $g_{ij,t}$ will be bounded between zero and one. The novel feature of this learning mechanism is that it allows the investors to endogenously switch their beliefs and permits them to change the weights they place on past data, in response to new information. Investors are allowed to increase or decrease the value of the gain in times when their coefficient estimates are different from the recent past. This algorithm is similar to the endogenous learning mechanism originally developed in Gaus (2014). The comparative numerical results below are presented for the CGL algorithm, and the gain specification following (8). The estimation of the gain parameters for (6) and (8) are discussed below in section 5.1 below.

The use of this functional form for EGL can be motivated using the analysis of Marcet and Nicolini (2003). The authors model investors as switching between a constant-gain and a decreasing-gain algorithm to explain recurrent hyperinflations in the 1980s in several countries. The investors are assumed to switch to a constant-gain learning model if the forecast errors observed are relatively large. In the endogenous learning model proposed here, we model the investors are changing their gain in response to the size of their forecast errors.

5 Evaluation of the Models and Implications for Investor Expectations

The mechanics of these two models of expectations formation may be understood as follows: the CGL algorithm allows investors to allow for structural changes in the data. In addition, the EGL mechanism allows them to compensate for large deviations in observed coefficients. Consider an investor who is forecasting yields in March 2015; she will put less weight on observations from 2005 than on observations from 2010 under CGL. However, if she observes a large deviation in the coefficient realizations of March 2015 relative to the past year, the EGL mechanism will allow the investor to vary her weights on 2010 (and 2005) data in response to the deviation. This compensation may involve placing more or less weight on the past observations. In contrast, under the constant coefficients model, she will be placing the same weights on the observations from 2005 and 2010 as before.

There are three aspects of investor expectations that we will analyze. First, do investors form expectations about the safest assets (AAA-only) differently from assets with higher risk? Second, for a fixed yield maturity, how do investors form conditional forecasts over different forecasting horizons for these asset types? That is, do they hold their beliefs constant while making forecasts over the short- and medium-term, or do the beliefs depend on the forecasting horizon? Finally, when the forecasting horizon is held constant, do investors keep their beliefs constant while making forecasts about the one- and ten-year yields, or are these beliefs varying? The results presented below will provide a framework for analyzing the beliefs of investors on these dimensions.

The models' forecasting performance is evaluated by comparing their root mean square errors (RMSEs), and we also discuss the implications of these results for modeling investor expectations. We use the full sample period available, from September 15, 2005 to June 8, 2015. The in-sample forecasts are constructed for the one-, five- and ten-year yields, at the one-, three- and six-month horizons for both types of yield curves. These horizons are set to match (on average) the number of trading days. For example, for constructing the one-month ahead forecast, the number of days is set at 21. We describe the computation of the optimal gains used in the different learning mechanisms below, and the model evaluations in section 5.2.

5.1 Determination of the Gain Parameters

The determination of the gain parameters under CGL has been a matter of significant research. Current estimates in the literature are available from Bayesian estimates from small-scale DSGE models (Milani, 2007) and by calibrating these parameters by matching the moments of forecast errors implied by the model and those of survey data. In this paper, we use the data to directly estimate optimal gain parameters. The estimation of the gain parameters are conditional on investors using the model of yield determination in (2a) and (5). Unlike previous analyses[10], we allow investors to use different gains for the four latent factors. Thus, the investors are no longer constrained to forming expectations of the level factor in the same way as for the slope factor. We also allow the gains to vary across forecasting horizons and asset types. The initial values of the gain parameters used are available upon request.

The optimization routine minimizes the root mean squared forecasting error (RMSE) between the actual yields and model-implied yield forecasts, over the parameters of the learning processes in (6) and (8). For the constant gain algorithm, this is g_i, for $i = \{0, 1, 2, 3\}$, and for the endogenous learning algorithm, k, $\bar{g}_i$ and $\bar{g}_i^{sf}$ for each the different factors. Optimal values of the parameters are estimated for each of the three forecasting horizons (1, 3 and 6 months). To our knowledge, our paper is the first to provide estimates of the gain parameter, using macroeconomic data observed at a daily frequency and varying forecast horizons. Conditional forecasts of the term structure of yields are then constructed from (7), using the optimal gains derived for the different forecasting horizons.

The values of the gain parameter are central to characterizing expectations using these learning models. The values of the gain parameter presented in tables 2, 3 and 4. The main observation is that across asset types, there is significant variation in the scaling factor. That is, investors appear to be adjusting the gain parameters at this frequency. For example, for the level factor β_0, at the 1-month forecasting horizon, investor beliefs place less weight on past observations in response to deviations for the AAA bonds, while for all-bonds portfolio, investors place more weight on past observations when such deviations are observed. Thus, the investors are placing more or less attention to past data, depending on the yield curve factor and the type of asset. Even though the gains are estimated based on daily data, it is

[10] An example is Laubach, Tetlow and Williams (2007).

noticeable that the optimization routine predicts such variation in the gains. This variation in conditional expectations would not be captured by a rational expectations model of investor beliefs; our results suggest that incorporating time-varying beliefs are essential to modeling financial market expectations.

Another broad result is that agents appear to be more "rational" over the longer yields in the sense that estimates of the constant gain and endogenous gain imply that the constant coefficient model is being used for some of the yield curve factors. Even though a rational expectations model cannot explain forecasts of the yield curve, certain aspects of the yield curve do appear to be explained by a rational expectations model. Taking a closer look at the 10-year yield in Table 4 we can see a well defined pattern: five of the six constant gains on β_0 are driven toward zero. This makes sense since this factor is considered the level, which our agents would associate with the central bank interest rates. The factor with the consistently highest constant gain values is β_1, which is associated with the slope. Again, this factor has the biggest impact in correctly forecasting the longer horizons.

5.2 Model Evaluation and Interpretation

Table 5 presents the comparison of conditional forecasts of the constant gain and endogenous learning models at the different forecasting horizons, risk profiles and yield maturities. We find that the largest gains in forecasting performance of the EGL mechanism, with respect to the CGL is found for the AAA-rated bonds for the 1-year maturity at the 1- and 3-month forecast horizons. Thus, compensating for deviations in observed coefficients with respect to the past observations appears to be more significant for the relatively riskless assets. This may be due to the composition of the AAA bond portfolio: investors who have lower risk tolerance are more sensitive to variations in the coefficients, and adjust their forecasting model accordingly. These results suggest that if investors respond more significantly to deviations in yields on safe assets (potentially adjusting their holdings of these assets as well), policy makers may be able to focus their initiatives on reducing variability in safe yields, instead of targeting a variety of assets with higher levels of riskiness.

In addition, we make the following observations. First, at longer maturities, the EGL improves upon the forecasting performance of CGL for "All yields". This is observed across forecasting horizons. For example, for the 10-year yield, at the 3-month horizon, the improvement of EGL relative to CGL is approximately 15%. Second, for the longest yield in our data (10-years), the gain in forecasting performance by using EGL increases as the forecast horizon increases for AAA-bonds. This is not observed for "All yields". Finally, for a fixed forecasting horizon, EGL implies similar forecasting performance as CGL for "All yields", when considered across yield maturities. This is not observed for AAA-bonds. These results suggest that there are important considerations for allowing for differential expectations formation processes for AAA- and All-bonds, due to their risk characteristics.

To further understand our results, we plot the values of the endogenous gains for the 1-month ahead forecasts of the 1-year maturity bonds for both bond pools in Figures 2(a) and 2(b). While both display time variation of the gains over time, the values for the riskier bond pool display larger movements. This reflects the greater variability of the underlying yield curve factors, which leads to poorer forecasts. Hence, a constant gain may serve just as well as an endogenous gain. In contrast, the AAA-rated bonds exhibit smoother transitions between the values of the gains. This suggests that risk averse investors monitor (or should monitor) the relationships between the underlying factors and gradually adjust how much they respond to "shocks" in the data.

In our view, the above results suggest the following implications. First, a large literature has used constant gain learning to model investor beliefs in theoretical frameworks. While this framework does well, our analysis suggests that during periods of large deviations from the historical average, it may not be insufficient for capturing the beliefs formation process. Adopting the endogenous learning algorithms proposed above provides an intuitive manner to model investor beliefs which can account for these deviations. Our results across the asset types suggest that the riskiness of an asset affects the manner in which beliefs are formed by investors, and presents an additional dimension that may be utilized by policy makers.

6 Conclusion

An empirical analysis of how subjective expectations evolve is useful for both macroeconomists and financial economists. This paper attempts to estimate how investors form conditional forecasts for safe assets relative to assets with higher risk. While estimating the optimal process to characterize conditional forecasts of investors, our methodology allows investors to vary how much weight they place on historical data while forecasting across asset types, maturities and forecast horizons. Our results for the Euro-area yield curves suggest that the risk profile of assets is an important characteristic for investors while forming conditional forecasts of yields (across maturities and forecast horizons). Future research will explore whether these differences in forecasting models for assets with different risk attributes is relevant for other datasets as well.

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Tables

Table 1: Moments of the Nominal Yield Curves for the Euro-Area

	Yield Curve Moments					
		AAA			All	
Moment	1 year	5 years	10 years	1 year	5 years	10 years
	September 2004 - December 2007					
Mean	3.0771	3.4655	3.8091	3.0816	3.4871	3.8565
St. Dev	0.7880	0.5385	0.3850	0.7911	0.5400	0.3887
	January 2008 - May 2015					
Mean	0.7922	1.7512	2.6874	1.2997	2.4634	3.4349
St. Dev	1.1992	1.1961	1.1391	1.1249	1.0767	1.0053

Note: The above moments are shown for end of month data on the fitted curves for the 2004-2015 data on AAA- and All-rated bonds.

Table 2: Optimal Values of the Gain Parameter for 1-year yield

Optimal Values of Gain Parameters								
	AAA				All			
Factors	CGL	EGL			CGL	EGL		
		$\bar{g}$	$\bar{g}^{sf}$	k		$\bar{g}$	$\bar{g}^{sf}$	k
Forecasting horizon $h = 1$ month								
β_0	0.0774	0.0001	0.0125	9	0.0048	0.0218	-0.0162	18
β_1	0.0796	0.0195	-0.0195	9	0.0001	0.0127	-0.0127	18
β_2	0.0161	0.0122	-0.0122	9	0.0099	0.2865	-0.2317	18
β_3	0.0898	0.0259	0.0101	9	0.0097	0.0536	0.2342	18
Forecasting horizon $h = 3$ months								
β_0	0.0019	0.0039	0.0039	60	0.0151	0.0001	0.0039	48
β_1	0.0001	0.0087	0.0081	60	0.0001	0.0114	-0.0114	48
β_2	0.0004	0.0009	0.0009	60	0.0255	0.0145	-0.0145	48
β_3	0.0347	0.0380	0.0182	60	0.0250	0.0111	-0.0067	48
Forecasting horizon $h = 6$ months								
β_0	0.0950	0.1056	-0.0212	121	0.0800	0.1325	0.0126	120
β_1	0.0999	0.1002	-0.0093	121	0.0001	0.0097	-0.0097	120
β_2	0.0842	0.1247	-0.0346	121	0.0884	0.0710	0.2041	120
β_3	0.0926	0.0858	0.0073	121	0.0919	0.2738	-0.2017	120

Note: These are the optimal gain values for constant gain (CGL) and endogenous gain with the scaling factor (EGL2), at the one-month forecasting horizon for the one-year yield, for the two types of bond holdings.

Table 3: Optimal Values of the Gain Parameter for 5-year yield

Optimal Values of Gain Parameters								
	AAA				All			
Factors	CGL	EGL			CGL	EGL		
		$\bar{g}$	$\bar{g}^{sf}$	k		$\bar{g}$	$\bar{g}^{sf}$	k
Forecasting horizon $h = 1$ month								
β_0	0.0019	0.0001	0.0038	19	0.0001	0.1294	-0.0516	20
β_1	0.0001	0.0288	-0.0287	19	0.0001	0.1127	0.0623	20
β_2	0.0002	0.0004	0.0001	19	0.0005	0.0836	0.0808	20
β_3	0.0585	0.0253	0.0148	19	0.0026	0.0704	0.0872	20
Forecasting horizon $h = 3$ months								
β_0	0.0012	0.0034	-0.0034	52	0.0001	0.1462	-0.0237	55
β_1	0.0001	0.0083	-0.0073	52	0.0001	0.1713	0.0132	55
β_2	0.0001	0.0001	0.0000	52	0.0035	0.1093	0.1232	55
β_3	0.0432	0.0001	0.0492	52	0.0028	0.2141	-0.0864	55
Forecasting horizon $h = 6$ months								
β_0	0.0018	0.1131	-0.0226	116	0.0438	0.1458	-0.0633	122
β_1	0.0009	0.1113	-0.0023	116	0.0294	0.1398	0.0468	122
β_2	0.0001	0.1017	0.0081	116	0.0391	0.0008	0.1964	122
β_3	0.0545	0.1051	-0.0045	116	0.0517	0.1907	-0.1612	122

Note: These are the optimal gain values for constant gain (CGL) and endogenous gain with the scaling factor (EGL2), at the one-month forecasting horizon for the five-year yield, for the two types of bond holdings.

Table 4: Optimal Values of the Gain Parameter for 10-year yield

	Optimal Values of Gain Parameters							
	AAA				All			
Factors	CGL	EGL			CGL	EGL		
		$\bar{g}$	$\bar{g}^{sf}$	k		$\bar{g}$	$\bar{g}^{sf}$	k
Forecasting horizon $h = 1$ month								
β_0	0.0001	0.0007	0.0012	11	0.0258	0.0001	0.0042	20
β_1	0.0013	0.0019	0.0309	11	0.1735	0.1922	-0.0442	20
β_2	0.0009	0.0349	0.0880	11	0.1705	0.0127	-0.0126	20
β_3	0.0010	0.0276	0.0490	11	0.1717	0.0095	-0.0094	20
Forecasting horizon $h = 3$ months								
β_0	0.0001	0.0001	0.0028	53	0.0001	0.0001	0.0025	56
β_1	0.0012	0.0064	-0.0051	53	0.0015	0.0421	-0.0321	56
β_2	0.0006	0.0002	0.0032	53	0.0001	0.0001	0.0000	56
β_3	0.0009	0.0096	-0.0081	53	0.0007	0.0001	0.0009	56
Forecasting horizon $h = 6$ months								
β_0	0.0001	0.0006	0.0004	112	0.0001	0.0001	0.0016	116
β_1	0.1068	0.0001	0.0022	112	0.0008	0.0001	0.0028	116
β_2	0.0001	0.0001	0.0000	112	0.0029	0.0001	0.0072	116
β_3	0.0021	0.0001	0.0041	112	0.0014	0.0001	0.0029	116

Note: These are the optimal gain values for constant gain (CGL) and endogenous gain with the scaling factor (EGL2), at the one-month forecasting horizon for the ten-year yield, for the two types of bond holdings.

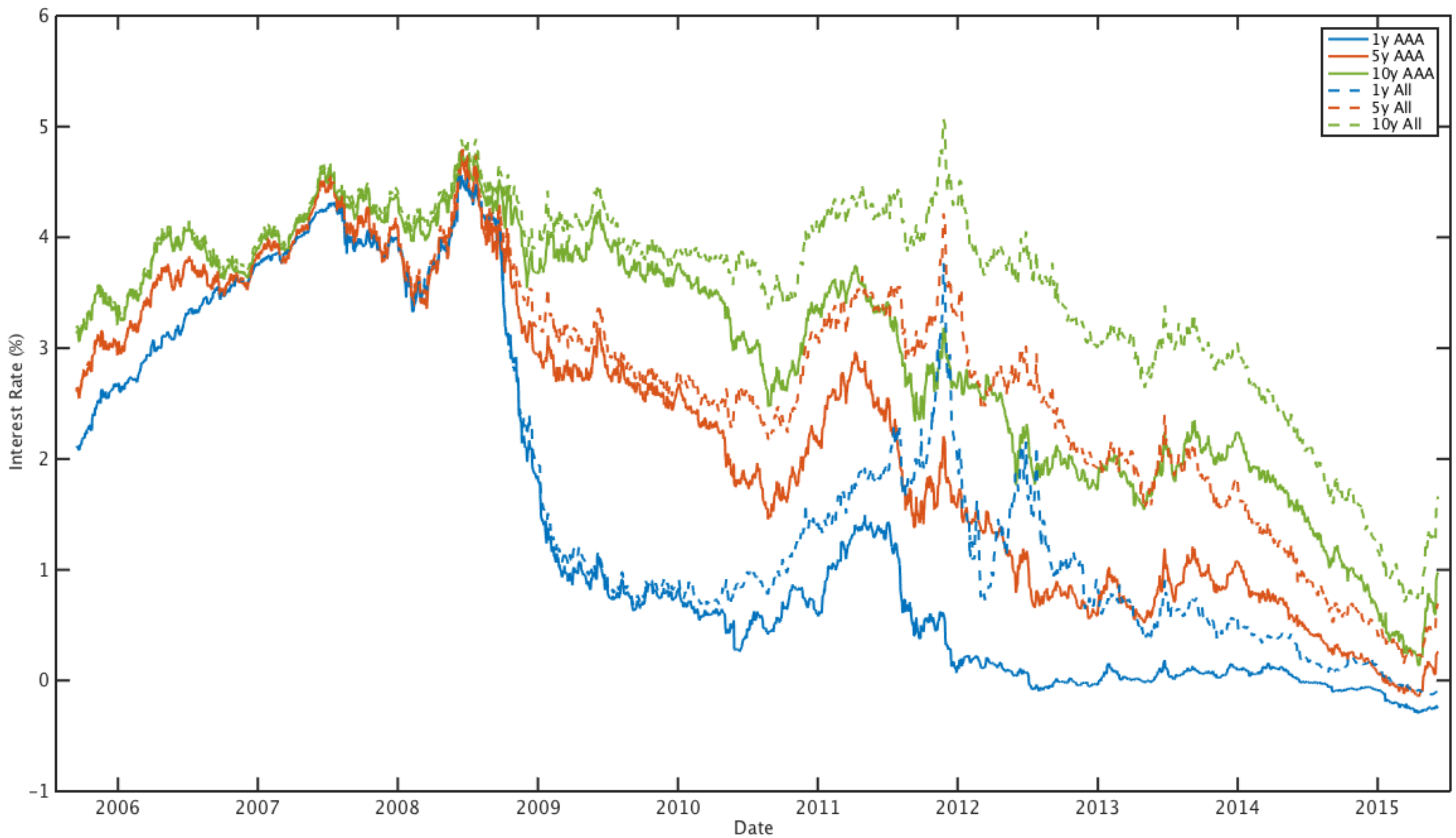
Table 5: Evaluating the Conditional Forecasts at 1-month horizon

Yield	AAA		All	
Maturity	RMSE-CGL	RMSE-EGL	RMSE-CGL	RMSE-EGL
	Forecasting horizon $h = 1$ month			
1 year	3.0562	1.8148	2.1366	1.9976
5 years	1.0521	0.8798	1.5670	1.6289
10 years	0.5746	0.5747	1.2127	0.9853
	Forecasting horizon $h = 3$ months			
1 year	1.9142	1.1594	2.1524	1.9366
5 years	1.0547	0.9032	1.5948	1.6385
10 years	0.4696	0.4727	1.0175	0.8489
	Forecasting horizon $h = 6$ months			
1 year	2.0235	2.6783	2.1242	1.9160
5 years	1.0667	1.1484	1.6354	1.6348
10 years	0.4832	0.4462	1.0521	0.9822

Note: These are the root mean square (RMSE) values for constant gain (CGL), endogenous learning (EGL) models, at the 1-month forecasting horizons for the three yield maturities and types of bond holdings.

7 Figures

Figure 1: Nominal Yield Curves for the Euro-Area

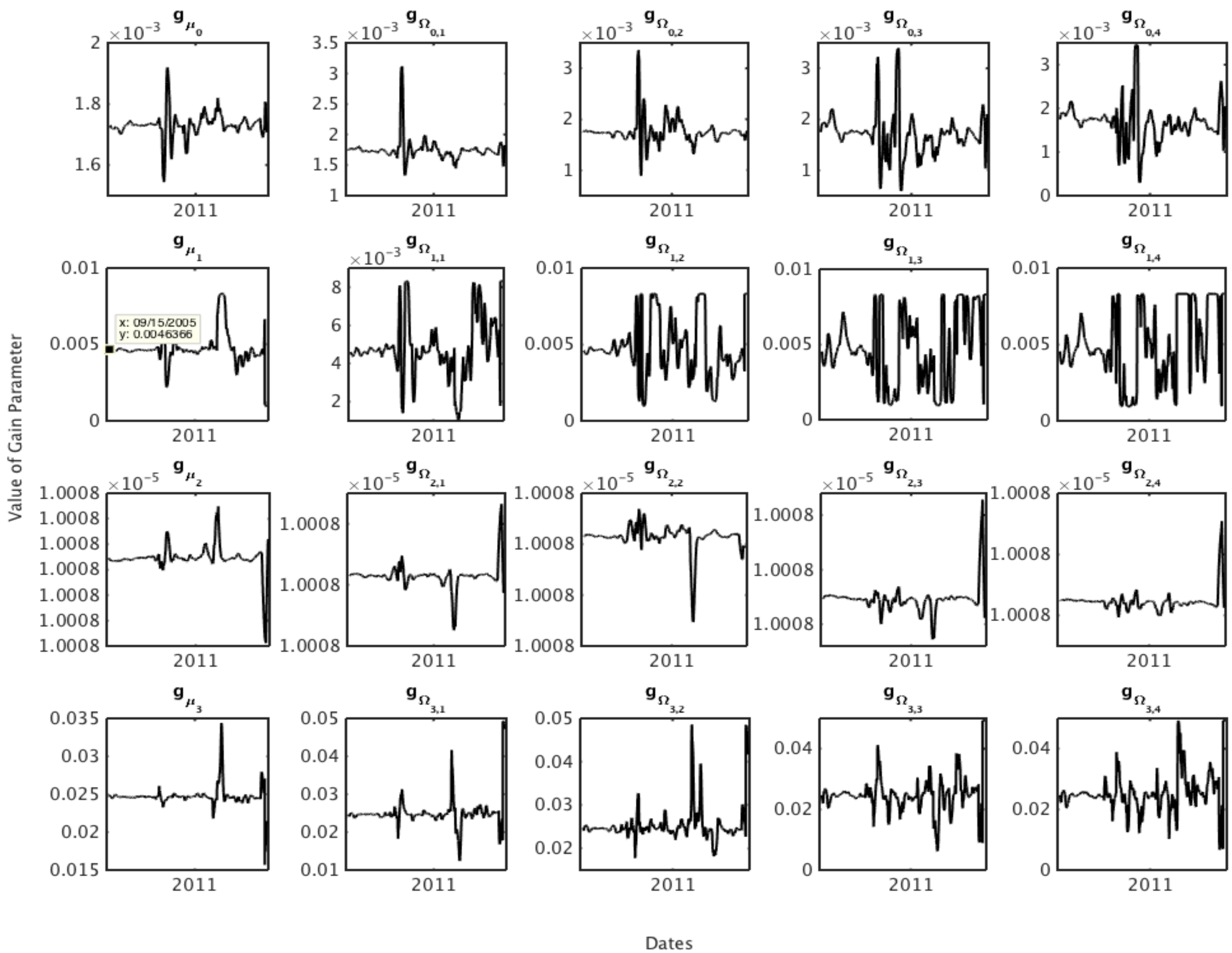


Note: The figure shows the evolution of the 1-, 5- and 10-year yields for AAA- and All-bonds. The solid lines show the AAA-yields, and the dashed lines show the All-bonds.

Figure 2: Evolution of Gains for 1-year yields at the 1-month horizon

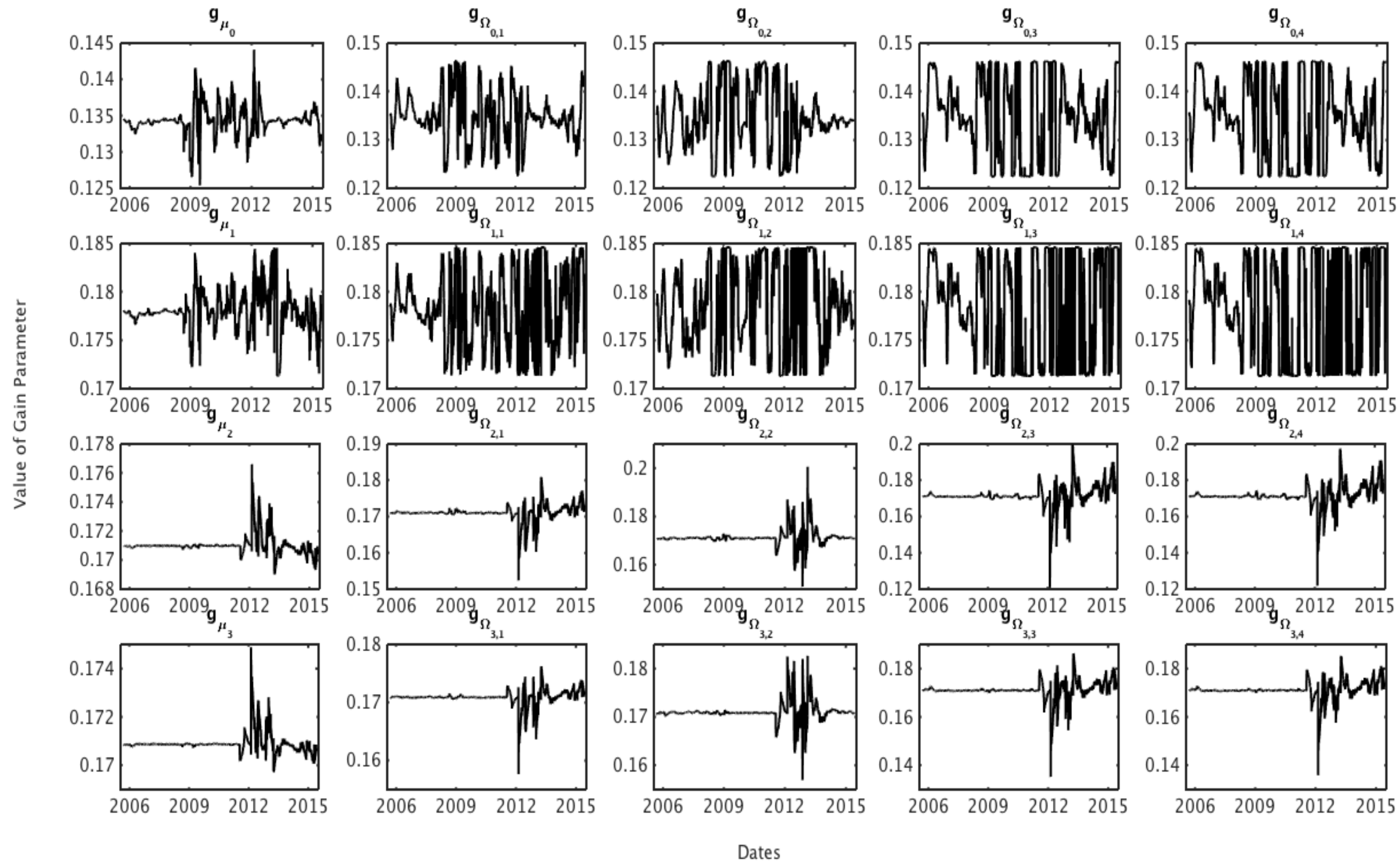
(*2a*)

AAA-rated bonds



(2b)

AAA- and other Bonds



Note: The figure shows the evolution of endogenous gains for the four latent factors (rows 1-4), and their dependence on a constant and past values of the factors themselves (columns 1-5).